MATH 140A Review: Set builder notation in $\mathbb{R}$

Simplify the notation:

1. $\mathbb{N} \cap [0, 1.2)$

> **Solution:** We have that $x \in \mathbb{N} \cap [0, 1.2)$ if and only if $x \in \mathbb{N}$ and $x \in [0, 1.2)$. Hence, $x \in \mathbb{N} \cap [0, 1.2) = 1$.

2. $\{x \in \mathbb{R} : x > 2\} \cap (-\infty, 8]$

> **Solution:** We have that $y \in \{x \in \mathbb{R} : x > 2\} \cap (-\infty, 8]$ if and only if $y \in \{x \in \mathbb{R} : x > 2\} = (2, \infty)$ and $y \in (-\infty, 8]$. Thus, $\{x \in \mathbb{R} : x > 2\} \cap (-\infty, 8] = (2, 8]$.

3. $\mathbb{R}^{\mathsf{c}}$

> **Solution:** We will show that the set is empty by way of contradiction. Assume that there exists a real number $x \in \mathbb{R}^{\mathsf{c}}$. Then, $x \notin \mathbb{R}$. This is a contradiction since $\mathbb{R}$ is the set of all real numbers, but x is a real number not in $\mathbb{R}$. Thus, the set $\mathbb{R}^{\mathsf{c}}$ is empty. That is, $\mathbb{R}^{\mathsf{c}} = \varnothing$.

4. $\mathbb{Z} \cap \mathbb{Q}^{\mathsf{c}}$

> **Solution:** We will show that the set is empty by way of contradiction. Assume that there exists $x \in \mathbb{Z} \cap \mathbb{Q}^{\mathsf{c}}$. Then, $x \in \mathbb{Z}$ and $x \in \mathbb{Q}^{\mathsf{c}}$. Since $x \in \mathbb{Q}^{\mathsf{c}}$, then x is irrational. This is a contradiction since $x \in \mathbb{Z}$, that is, x is an integer number (which is also a rational number). Thus, the set $\mathbb{Z} \cap \mathbb{Q}^{\mathsf{c}}$ is empty. That is, $\mathbb{Z} \cap \mathbb{Q}^{\mathsf{c}} = \varnothing$.